Study on the Volatility Smile of EUR/USD Currency Options and Trading Strategies

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Introduction

Statement of the problem

According to Black-Scholes-Merton model, the probability distributions of asset prices are lognormal. Each option price has an implied volatility. On foreign exchange markets, the volatility of an exchange rate is not the same at different strike price and the price of the exchange do not always change smoothly.

An option is priced using volatility depending on its strike price and time to maturity. A plot of the implied volatility of an option as a function of its strike price is known as a volatility smile.

The implied volatility of foreign currency options is lower for at-the-money options, and become higher as the option move to in-the-money or out-of-the-money. In other words, implied distribution of asset return has heavier tails than the lognormal distribution as Black-Scholes model suggested which has the same mean and standard deviation.

Also, the empirical results show that the volatility smiles are not always remaining constant. What happened if the shapes of the volatility smile changes? What does the change mean to option? And what is the difference among different maturities? In this paper, we are going to conduct an empirical study on the volatility surface through two dimensions. First, we focus on the slope of the volatility smile and the impact on the option price. Secondly, we test the performance of various trading
strategies concerning the signaling from slope changes, including directional trading, delta-neutral portfolio and double-traded delta-neutral portfolio. The idea is to try to utilize the signals from volatility smile and catch profit through corresponding trading strategies.

**Literature review**

Unlike equity option market, foreign exchange option market has different convention of option price and strikes. The dollar price and strikes are not observable in the market. Instead, it is implied volatility and delta strikes that represent the option information. Therefore, in the data collecting period, we are facing the volatility surface and smiles as raw data. Through the “Volatility surfaces: theory, rules of thumb, and empirical evidence,” (Daglish, T., J. Hull, and W. Suo, 2007) and “FX Volatility Smile Construction” (Dimitri Reiswich, and Uwe Wystup, 2010), we understand the construction rules for volatility surface and how we can convert this convention back to dollar sign which we would be easily to use for strategies testing.

“Jump risk, stock returns, and slope of implied volatility smile, Shu Yan, Moore School of Business, University of South Carolina, Columbia, SC 29208, United States” suggested that expected stock return will decrease when stock jumps as empirical evidence exists for jumps in stock prices. And the jump size can be represented by the slope of the option implied volatility smile. The theses suggested that the slope may predict future stock returns. Portfolio with low slope may generate higher returns than portfolio with high slope. 1.9% monthly profit is generated by buying
the lowest slope portfolio and shorting the highest slope portfolio generates.

“The implied volatility term structure of stock index options, Scott Mixon, Bates White, LLC, 1300 Eye Street NW, Suite 600, Washington, DC 20005, United States” suggested that the slope of the term structure of option volatility can forecast future short dated implied volatility. The hypothesis forecast works better with a volatility risk premium term. The theses suggested that a portfolio can generate return by selling volatility to the ones who believe the risk premium interpretation to capture the gap between implied and realized volatility.

Objectives of the study

Throughout the study, based on the empirical evidence we are dedicated to achieve three objectives: 1) the general shape of the volatility smiles of EUR/USD currency options 2) under the volatility surface, how different strategies perform in general.

First, we are going to generate the volatility surface and identify the shape of volatility smiles of EUR/USD currency options of different time maturities and see whether they are in U-shape or upward/downward sloping in general, through the stream of historical options prices. Secondly, we are trying to identify the slope between two options and explore the signals of change of slope. We would like to know whether those slopes are changing constantly around an ‘average’ level or has some obvious trend through different periods. Lastly, we are going to test the trading strategies of directional trading, delta-neutral portfolio trading and double
delta-neutral portfolio trading based on the volatility slope change. The historical average return of those strategies based on our volatility smiles and slope changes will tell us whether we can catch the profit or loss from signals.

Data and methodology

Data Source

We retrieved our data from Bloomberg terminal and other online data providers to have EUR/USD historical exchange rate, deposit rate for EUR and USD separately, option implied volatility and implied volatility surfaces for EUR/USD options. The time span crosses past 10 years from April 1, 2004 to April 1, 2014. We collected data for every week namely every 1st, 8th, 15th and 23rd of the month for convenience. Totally, we have 481 sets of data. One point to address is before 2007, the EUR/USD option traded in OTC market was of low volume, i.e. low liquidity. Therefore, the data sets from the early years were not change a lot since there may be less trading at the time.

For detail, we are using all one-month maturity options, annual deposit rate for USD and EUR separately from bank average rate of certificate of deposit. The original call/put strike/premium is quoted in delta/implied volatility convention from Bloomberg Terminal. After we draw all volatility surfaces, we picked the 25D and ATM options as they are actively traded most of the time and liquidity risk is low.
which is good for our research to avoid the interference from high risk. We also retrieved historical exchange rate (spot rate) of EUR/USD as we would use it to calculate the dollar amount of price and strike and the spot is also involved in our strategy tests.

Conventions & price calculation

The market conventions for options are using delta for strikes and implied volatility for prices. In our study, our raw data from Bloomberg are all quoted in that convention and therefore we need calculate the original dollar price and strikes out of the delta and implied volatility. As the volatility surface from Bloomberg is based on the Black-Scholes Model, our calculation is literally applying the reverse function of $N(d_1)$ to get the strike prices and then prices are calculated from the model afterwards.

Black-Scholes Model for call and put:

$$c = S_0 e^{-rt} N(d_1) - K e^{-qt} N(d_2)$$

$$p = K e^{-rt} N(-d_2) - S_0 e^{-qt} N(-d_1)$$

$$d_1 = \frac{\ln \left( \frac{S_0}{K} \right) + \left( r - q + \frac{\delta^2}{2} \right) t}{\delta \sqrt{t}}$$

$$d_2 = d_1 - \delta \sqrt{t}$$

The inverse calculation process utilizes the NORMSINV function in Excel, or inverse
function of cumulative normal distribution function to return the true d1 from cumulative standard normal distribution and the formula for K calculation is as following:

\[
K(\text{call}) = Se^{-\text{NORMSINV}(\Delta e^{(r-q)t})\delta\sqrt{t}+(q+\frac{\delta^2}{2})t}
\]

\[
K(\text{put}) = Se^{\text{NORMSINV}(\Delta e^{(r-q)t})\delta\sqrt{t}+(q-\frac{\delta^2}{2})t}
\]

As \( r \) is the domestic deposit rate (USD), \( q \) is the foreign deposit rate (EURO), \( \delta \) is the implied volatility, \( S \) for spot rate and \( t \) is the time to maturity.

Then using the BS Model, we can easily calculate the option premium out of above factors.

**Volatility Smile**

Using 10 years’ monthly average volatility of EUR/USD option which expired in 1 month, we plot the following chart for volatility smile. The skewness is observable and is due to the interest rate effect of EUR and USD currencies. During the last 10 years’, the interest rate in Europe is higher than that in US most of the times, so the lowest point is in the right part in the volatility smile.
Slope

We define slope as the difference of volatility between 25D Put and ATM Put divide by the volatility of ATM Put. \( \frac{(\text{Vol } 25\text{D Put}) - (\text{Vol ATM Put})}{\text{Vol ATM Put}} \). In this way, we set ATM Vol as the benchmark so that we may investigate the relative volatility of 25D Put comparing to ATM Put.

We use put option as the primary data as we can see from the following chart that implied volatility draw from put option are more of normal distribution than implied volatility draw from call option.
We use 25D Put option volatility in this research. We can see from the following chart about the slope change in every month that the slope change of 25D options is more of normal distribution.
As volatility change in a certain range, we believe it has the characteristic of mean reversion.

**Delta Neutral**

Delta is “The ratio comparing the change in the price of the underlying asset to the corresponding change in the price of a derivative.” A delta neutral (Delta = 0) portfolio’s value will not be affected by the underlying assets.

In this case, we trade 25D Put and ATM Put to construct the delta neutral portfolio. For 25D Put, the delta equals 0.25; for ATM Put, the delta equals 0.5. As a result, if we would like to long volatility at 25D Put, we may long 2 25 D Put and simultaneously short 1 ATM Put. Delta of the portfolio equals $2 \times 0.25 - 0.5 = 0$. On the contrary, if we would like to short volatility at 25D Put, we may short 2 25 D Put and simultaneously long 1 ATM Put. Delta of the portfolio equals $-2 \times 0.25 + 0.5 = 0$. 

![25D Put Slope Change](image-url)
Trading Strategies

Strategy 1

Methodology

First, we use the information from the change of slope in volatility smile to bet on the spot of EUR/USD rate.

We tried two strategies to bet on the spot of the EUR/USD rate: (1) If the slope of volatility smile measured by 25D Put Option and ATM Option increased this month, we short USD spot and long EUR Spot. (2) If the slope of volatility smile measured by 25D Put Option and ATM Option increased this month, we short EUR spot and long USD Spot.

As volatility is always described as the fear of investor, the price of a security is predicted to drop if the volatility of that security increase. Such relationship is proved to be true between S&P500 and VIX. We would like to test the relationship on EUR/USD to see whether the spot of one currency is going to drop if the volatility of that currency increases.

The first strategy is profitable if most of the volatility in EUR/USD is contributed by USD. The second strategy is profitable if most of the volatility in EUR/USD is contributed by EUR. Neither of the strategy is profitable if the volatility in EUR/USD is
contributed by both USD and EUR.

Result

Both of the strategies do not earn a normal profit. Both strategies will generate a loss or have a marginal profit.

We believe the reason is that the volatility in EUR/USD is contributed by both USD and EUR. As a result, trading spot based on the change of volatility smile is a pure guess with 50% change to win as one never know whether currency contributes more volatility in the volatility smile next month. Or fundamental analysis may be needed to better facilitate this trading strategy.
Above are the Monthly Return of EUR/USD Spot and Slope Change of EUR/USD Volatility Smile. We can see that there is no obvious relationship in between which proves our result above.

**Before Strategy 2 and Strategy 3**

Other than trading spot, we may also trade volatility based on the change in volatility smile. As there is no volatility index future for EUR/USD, we first assume we may trade EUR/USD Historical Volatility Index to see whether the change in volatility smile has the forecasting ability in future volatility.

The following chart is the 5-year EUR/USD Historical Volatility Index.
If we long the index if the slope change is larger than 0.03 and short the index if the slope change is smaller than -0.03 at the beginning of every month, the monthly profits and compounding profits is shown in the charts below.
Under the assumption that we can trade EUR/USD Historical Volatility Index, it shows that the trading strategy based on mean reversion is profitable. It generates 96.41% return for 10 years which is equivalent to 14.45% annual yield.

After proving trading the change of slope of volatility is profitable, we need to find out how to realize the strategy by using the financial products we may trade, for instance, foreign currency options.

**Strategy 2**

**Methodology**

In strategy two, we are testing the delta-neutral portfolio that catches the profit from abnormal deviation from average slope. The idea is based on the mean reversion process of implied volatility slope between two options with different strikes, in our case, ATM put and 25D put.
First is the delta-neutral portfolio construction with two long positions of 25D put and one short position of ATM put. Secondly, the trigger of trading is the change of slope of implied volatility between the options. According to mean reversion idea applied broadly in financial world, we believe whenever the slope becomes larger, we expect it to become smaller, and back to normal in the following period. Then we short the portfolio in order to catch the trend. In our research we tested in strategy two that the change of slope will be recovered immediately in the period (one month) after and we trade only one time per change no matter how big it changes. And it is not influenced by the change of underlying asset, i.e. spot price. Similarly, if the slope becomes far smaller than average, we trade oppositely with long position.

The cost of the portfolio is calculated as following: 1) long position, we calculate the premium paid directly as cost of strategy. 2) Short position, we follow the market convention of margin requirement to calculate the delta margin and vega margin. And we use the margin required as the cost. For the delta margin, since we use the delta-neutral portfolio, the delta margin requirement is zero at trading time. For vega margin, we calculate the vega exposure first from the premium, strikes and spot price. Then we use the 11% margin requirement convention for one-month maturity option. In our strategy three, we use the same convention to calculate the cost.

Margin Requirement = Delta Margin + 100 × Vega Exposure × IV × 11%
Vega: \[ Se^{-q\tau} \Phi(d_1)\sqrt{\tau} = Ke^{-r\tau} \Phi(d_2)\sqrt{\tau} \]

Delta: \[ -e^{-q\tau} \Phi(-d_1) \]

Then we repeat the strategy every week in the past ten years for 1-month maturity EUR/USD options. And calculate the return and cumulative return sequentially. The result is showed in the next part of this paper.

**Result**

We tested the strategy 2 to our ten year time span with weekly traded option portfolio of one-month maturity. In the result, we found that only from the year 2007, trading executions appeared frequently and compounding rate of return of 10 years is only 88.76%, or 6.56% annually. From Jan.1.2007 to Jan.1.2014, in seven years’ time, the cumulative rate of return is 119.27% or annually 11.87%.

As we analysis further into the result, we found:

1. The reason behind the fact that there is no trading execution before 2007 is that at the time, people didn’t trade much on EUR/USD option. The volume was low and liquidity is much lower, especially for the OTC market.

2. The rate of return is not as high as our expected. The reason is that the OTC market now requires substantial amount of margin. The high vega margin
requirement actually drag down our overall performance. If we ignore the 11% vega margin with pure profit and loss, the internal rate of return through ten years is 74.37% per annum.

3. The scale of slope change changes significantly before the crisis, during the crisis and post crisis. When we tested the trigger of execution, we found that the slope change appeared to be very large during 2007 and 2008 and the slope barely changes before 2007 due to low volume. However after 2008, the volume of trading is high, but the slope changes little from month to month. And as a result, how we set the trigger, i.e. the ‘significant’ slope change, makes a difference for our trading performance.

4. Normality of returns. When applying the normality test, four moments are 0.201% mean, 3.78% variance, 1.62 skewness and 14.90 kurtosis. It means that the distribution of returns is skewed to the right with fatter tails and less risky of extreme values. One point to mention is that this distribution shows the return of our strategy including the periods that we choose not to trade.
Strategy 3

Methodology

In strategy 3, we follow most of the steps in strategy 2 except we will trade more if the slope of volatility smile changes more. We still hold delta-neutral portfolio and trade based on the mean reversion process of implied volatility slope.

We tested in strategy three that the change of slope will be recovered immediately in the period (one month) after and we trade only one time per change no matter how big it changes. Then we repeat the strategy every week in the past ten years for 1-month maturity EUR/USD options. And calculate the return and cumulative return sequentially. The result is showed in the next part of this paper.

In strategy 2, we short 2 25D Put Option and long 1 ATM Option when the slope increase; we long 2 25D Put Option and short 1 ATM Option when the slope decrease.

In strategy 3, we will require tougher condition to trigger the trade. In strategy 2, we trade no matter how big it changes while in strategy 3, we trade only when the change of slope is larger than 0.02. In strategy 3, we trade double portion of options if the change of slope is larger than 0.02. We short 4 25D Put Option and long 2 ATM Option when the slope increase by more than 0.02; we long 4 25D Put Option and short 2 ATM Option when the slope decrease by more than 0.02. We short 2 25D Put Option and long 1 ATM Option when the slope increase by more than 0.01; we long
2. 25D Put Option and short 1 ATM Option when the slope decrease by more than 0.01.

We follow the same cost and margin requirement calculation method as in strategy 2.

**Result**

In strategy 3, in ten year time span with weekly traded option portfolio of one-month maturity, we found that compounding rate of return of 10 years is 129.54%, or 8.664% annually. From Jan.1.2007 to Jan.1.2014, in seven years’ time, the cumulative rate of return is 129.54% or annually 12.604%. During the crisis time which is 2007 to 2011, the strategy works the better, yielding 21.525% per annum.

As we analysis further into the result, we found:

(1) Same as strategy 2, the reason that there is no trading execution before 2007 is the low volume and liquidity in OTC market. The rate of return is not high because of high vega margin requirement. And our strategy has a better explaining power during the crisis time.

(2) When adapting tougher condition to trigger the trades, the return increases. It is reasonable because it shows that the explaining power is larger when the change in slope of volatility smile is larger.
Conclusion

This project is a study on the “Volatility Smile of EUR/USD Currency Options and Trading Strategies”. By adopting mean reversion methodology in EUR/USD currency option volatility and holding a delta neutral portfolio, we found the strategy profitable. The strategy works the best during the crisis times from 2007 to 2010 when it gave an annual return of 21.525%. It shows that the strategy has better explaining power when the option market is more volatile. In other words, the mean reversion phenomenon is more obvious when the change in volatility in previous term is larger.
Reference

[1] Shu Yan, *Jump risk, stock returns, and slope of implied volatility smile*, Moore School of Business, University of South Carolina, Columbia, SC 29208, United States

